Estimating extremal dependence using B-splines

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B-spline smoothing techniques are commonly used in functional data analysis. In this talk, I will explain how this tool can be adapted to derive intrinsic estimators of the Pickands dependence function characterizing the dependence in the maximum attractor of a bivariate continuous distribution. The approach is rooted in a rank-based transformation of the data due to Cormier et al. (2014). As shown therein, a plot of the transformed pairs of points provides a useful tool for detecting extreme-value dependence or extremal tail behavior. When the case arises, a constrained B-spline can be fitted through a suitable subset of the points to get an estimator of the Pickands dependence function associated with the extreme-value attractor. This estimator is intrinsic, i.e., it satisfies all the conditions required to qualify as a Pickands dependence function. The excellent finite-sample performance of this estimator was documented through simulations by Cormier et al. (ibid.).
As a follow-up to this work, I will state minimal conditions under which this estimator is consistent, and I will give its limiting distribution. This result is valid whatever the order of the B-splines. I will also demonstrate through theory and simulations that while B-splines of order 3 are sufficient to estimate the Pickands dependence function with accuracy, B-splines of order 4 are essential to grasp the features of the spectral distribution associated with the maximal attractor. This approach leads to an estimator that generally outperforms the maximum empirical likelihood estimator studied by Einmahl & Segers (2009). This talk is based on joint work with A. Bücher, C. Genest, and D. Sznajder.

[1] Cormier, E., Genest, C., Nešlehová, J.G., Using B-splines for nonparametric inference on bivariate extreme-value copulas, Extremes 17, 633-659, 2014.

[2] Einmahl, J.H., Segers, J., Maximum empirical likelihood estimation of the spectral measure of an extreme-value distribution, The Annals of Statistics 37, 2953-2989, 2009.