Quantile-based risk sharing

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We address the problem of risk sharing among agents using a two-parameter class of quantile-based risk measures, the so-called Range-Value-at-Risk (RVaR), as their preferences. The family of RVaR includes the Value-at-Risk (VaR) and the Expected Shortfall (ES), the two popular and competing regulatory risk measures, as special cases. We first present an inequality for RVaR aggregation, showing that a special form of subadditivity is satisfied by RVaR. Then, the risk sharing problem is solved by explicit construction. Three relevant issues in the optimal allocations are investigated: extra sources of randomness, comonotonicty, and model uncertainty. We show that in general, a robust optimal allocation exists if and only if none of the underlying risk measures is a VaR. Practical implications of the main results for risk management and policy makers are discussed, including gambling behaviour, moral hazard, regulatory arbitrage, and model misspecification. In particular, in the context of regulatory capital reduction, we provide some general guidelines on how a regulatory risk measure can lead to certain desirable or undesirable properties of risk sharing among firms, and show novel advantages of ES from the perspective of a regulator.