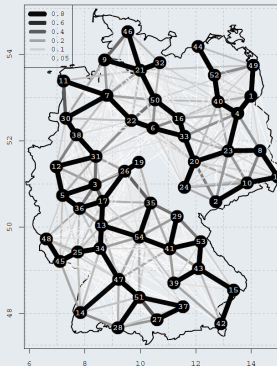
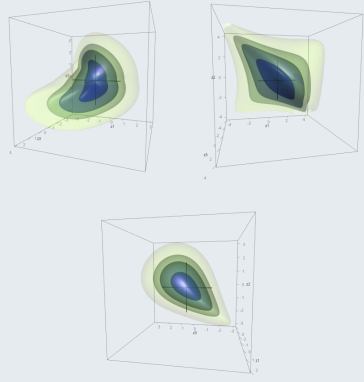




# Dependence Modeling in Finance, Insurance and Environmental Science

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Insurance  
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DEPENDENCE MODELING IN  
FINANCE, INSURANCE, AND  
ENVIRONMENTAL SCIENCE

MUNICH, MAY 17 – 19, 2016

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# Talks

## Financial Networks and Systemic Risk

*Kerstin Awiszus, Leibniz Universität Hannover & Center for Risk and Insurance, Hannover*

*Stefan Weber, Leibniz Universität Hannover & Center for Risk and Insurance, Hannover*

Systemic risk refers to the risk that a financial system is susceptible to failures due to the characteristics of the system itself. If strong links between financial institutions are present, a shock to only a small number of entities might propagate through the system and trigger substantial financial losses. Significant dependence can thus increase the risk of a system-wide breakdown. The talk presents a comprehensive model of a financial system that integrates the dependence of market participants through nominal liabilities, bankruptcy costs, fire sales, and cross-holdings. For the integrated financial market we prove the existence of a price-payment equilibrium and design an algorithm for the computation of the greatest and the least equilibrium. Systemic risk measures and the number of defaults corresponding to the greatest price-payment equilibrium are analyzed in several comparative case studies. These illustrate the individual and joint impact of the underlying factors.

## Vine copula based inference of multivariate event time data

*Nicole Barthel, Technische Universität München*

*Candida Geerdens, Universiteit Hasselt*

*Matthias Killiches, Technische Universität München*

*Paul Janssen, Universiteit Hasselt*

*Claudia Czado, Technische Universität München*

In many studies multivariate event time data are generated from clusters of equal size. Flexible models are needed to capture the possibly complex association pattern in such data. Vine copulas serve this purpose. Inference methods for vine copulas are available for complete data. Event time data, however, are often subject to right-censoring. As a consequence, the existing inferential tools, e.g. likelihood estimation, need to be adapted. We develop likelihood based inference for clustered right-censored event time data using vine copulas. Due to the right-censoring single and double integrals show up in the likelihood expression and numerical integration is needed for the likelihood evaluation. A simulation study for three-dimensional data

provides evidence for the good finite sample performance of the proposed method. Using a four-dimensional data set from veterinary medicine, we show how an appropriate vine copula model can be selected for the data at hand.

[1] Barthel, N., Geerdens, C., Killiches, M., Janssen, P., Czado, C., Vine copula based inference of multivariate event time data, submitted.

## Inferring Tail Correlation from Option Prices

*Carole Bernard, Grenoble Ecole de Management, France*

*Oleg Bondarenko, University of Illinois at Chicago, USA*

*Steven Vanduffel, Vrije Universiteit Brussel, Belgium*

We propose a new method (using [2] and [1]) to assess the interaction among assets in the market under the risk-neutral probability. Specifically, we exhibit dependence structures among stocks that are consistent with prices of options written on individual stocks as well as the corresponding index options. We show how the implied dependence obtained with our methodology improves upon the implied correlation index that was recently introduced by the Chicago Board Options Exchange (CBOE). In particular, our approach can measure implied tail correlations (e.g. correlations in a crisis scenario).

[1] Bernard C., Bondarenko, O. and S. Vanduffel (2015): “Rearrangement Algorithm and Maximum Entropy”, *Working paper*.

[2] Puccetti, G., and L. Rüschendorf (2012): “Computation of sharp bounds on the distribution of a function of dependent risks,” *Journal of Computational and Applied Mathematics*, 236(7), 1833-1840.

## Spatio-temporal synthesis of long continuous precipitation series in a high temporal resolution

*A. C. Callau Poduje, Institute of Water Resources Management, Hydrology and Agricultural Hydraulic Engineering, Faculty of Civil Engineering and Geodetic Science, Leibniz University Hannover, Germany (callau@iww.uni-hannover.de)*

*U. Haberlandt, Institute of Water Resources Management, Hydrology and Agricultural Hydraulic Engineering, Faculty of Civil Engineering and Geodetic Science, Leibniz University Hannover, Germany*

Long and continuous rain series are required for designing and evaluating urban drainage systems. However, the temporal and spatial resolution required for urban

hydrology, limit the available data to often short time series, therefore it is propitious to develop a precipitation model to allow for the generation of long synthetic series. A stochastic model is applied for this purpose which involves an alternating renewal process describing a system which can be in one of two possible states: wet or dry spells. Spells are described by their duration and amount of rainfall and are simulated stochastically. The generation of rainfall time series using this type of models is straight forward for single sites; therefore it is the aim to present an extension of the existing model to spatio-temporal simulations. The synthesis of several spatially correlated time series proposed here consists of a combination of a hybrid model with a multivariate vine copula. Rainfall series registered in several stations located in three cities in Germany are used to develop the proposed methodology. Vine copulas present an advantage of modeling multiple variables by pair-copulas which results in high flexibility to reproduce different dependence structures. Therefore this type of copulas was found to be an appropriate tool for simulating events in several stations, due to their capability of representing the joint behavior of rainfall characteristics for the different stations. Spatial consistency criteria resulting from long synthetic time series were compared with observations. The combination of the hybrid model with the vine copula shows to perform as a satisfactory extension of the model for multiple sites simulations.

## Vine regression

*Roger Cooke, Resources for the Future*

*Harry Joe, University of British Columbia, Canada*

*Bo Chang, University of British Columbia, Canada*

Regular vines or vine copula provide a rich class of multivariate densities with arbitrary one dimensional margins and Gaussian or non-Gaussian dependence structures. The density enables calculation of all conditional distributions, in particular, regression functions for any subset of variables conditional on any disjoint set of variables can be computed, either analytically or by simulation. Regular vines can be used to fit or smooth non-discrete multivariate data. The epicycles of regression - including/excluding covariates, interactions, higher order terms, multi collinearity, model fit, transformations, heteroscedasticity, bias, convergence, efficiency - are dispelled, and only the question of finding an adequate vine copula remains. This article illustrates vine regression with a data set from the National Longitudinal study of Youth relating breastfeeding to IQ. Based on the Gaussian C-Vine, the expected effects of breastfeeding on IQ depend on IQ, on the baseline level of breastfeeding, on the duration of additional breastfeeding and on the values of other covariates. A child given 2 weeks breastfeeding can expect to increase his/her IQ by 1.5 to 2 IQ points by adding 10 weeks of Breastfeeding, depending on values of other covariates. Av-



eraged over the NLSY data, 10 weeks additional breast feeding yields an expected gain in IQ of 0.726 IQ points. Such differentiated predictions cannot be obtained by regression models which are linear in the covariates.

## **Bootstrap percolation in inhomogeneous, directed random graphs and financial contagion**

*Nils Detering, Department of Mathematics, University of Munich*

*Thilo Meyer-Brandis, Department of Mathematics, University of Munich*

*Konstantinos Panagiotou, Department of Mathematics, University of Munich*

Bootstrap percolation is a process that is used to model the spread of an infection on a given graph. In the model considered each vertex is equipped with an individual threshold. As soon as the number of infected neighbors exceeds that threshold, the vertex gets infected as well and remains so forever. We perform a thorough analysis of bootstrap percolation on a novel model of directed and inhomogeneous random graphs, where the distribution of the edges is specified by assigning two distinct weights to each vertex, describing the tendency of it to receive edges from or to send edges to other vertices. Under the assumption that the limiting degree distribution of the graph is integrable we determine the typical fraction of infected vertices. Our model allows us to study settings that were outside the reach of current methods, in particular the prominent case in which the degree distribution has an unbounded variance. Among other results, we quantify the notion of "systemic risk", that is, to what extent local adverse shocks can propagate to large parts of the graph through a cascade, and discover novel features that make graphs prone/resilient to initially small infections. We show how our results can be used to study default contagion in a financial network.

## **Copula-based simultaneous test procedures with applications to stress testing**

*Thorsten Dickhaus, University of Bremen, Institute for Statistics*

We are concerned with simultaneous testing of a family of null hypotheses under a single statistical model. In this, we assume that the individual tests are carried out by means of (marginal)  $p$ -values and that these  $p$ -values, regarded as random variables, are dependent. The traditional type I error measure in multiple testing is the family-wise error rate (FWER). We express the threshold of an FWER-controlling simultaneous test procedure (STP) in the sense of [1] in terms of the copula function  $C_{\emptyset^*}$  of the family of  $p$ -values under the global hypothesis, assuming that each of

these  $p$ -values is marginally uniformly distributed on the unit interval under the corresponding null. This offers the opportunity to exploit the rich literature on copula-based modeling of multivariate dependency structures for the construction of STPs in non-Gaussian situations.

In the case of an unknown copula  $C_{\theta^*}$ , we derive confidence regions for the realized FWER of multiple tests which are empirically calibrated at a given (global) level of significance. To this end, we regard the FWER as a derived parameter of a multivariate parametric copula model. It turns out that the resulting confidence regions are typically very much concentrated around the target FWER level, while generic multiple tests with fixed thresholds are in general not FWER-exhausting.

Finally, we illustrate our theoretical results by considering a particular class of multiple test problems of practical relevance in detail, namely, multiple support tests. We apply these multiple tests to a stress testing scenario regarding exchange rate risks in finance. The presentation is based on [2].

- [1] Gabriel, K.R, Simultaneous test procedures - some theory of multiple comparisons, *Ann. Math. Stat.* 40, 224-250, 1969.
- [2] Stange, J, Bodnar, T, Dickhaus, T, Uncertainty quantification for the family-wise error rate in multivariate copula models, *AStA Adv. Stat. Anal.* 99, 281-310, 2015.

## Modelling dependence structure between stock indices using a second-order regime-switching vine copula

*Alex Donovan, University of Essex*

In this study first-order regime-switching vine copula is extended to a second-order regime-switching vine copula. This model is applied to returns of the S&P500, FTSE100 and DAX stock indices. Empirically, two regimes are identified: one of high and one of low dependence. In order to allow asymmetries in the upper and lower tail-dependence, we have compared 10 vine copulas with different tail behaviour. The dependence parameters are held constant within each regime. The standard errors of the estimates are computed using Godambe information matrix. For the sample period under investigation, based on the principle of maximum likelihood, Gaussian bivariate copulas were selected as main building blocks in the vine copula specification. The model is then compared against the benchmark model in which regime variable follows first-order process. The information criteria such as AIC and BIC suggest that the second-order regime-switching vine copula may be a better choice.

# From subcopulas to copulas

*Fabrizio Durante, Free University of Bozen–Bolzano, Italy*

According to Sklar's Theorem it is known that the probability law of any random vector can be expressed as a composition of the distribution functions of all one-dimensional margins and a suitable copula. However, while the copula associated with a random vector is unique when all margins are continuous, in the non-continuous case, various copulas can be associated with the same random vector, all being coincident in a subset of the copulas domain. This fact poses the natural question of how it is possible to construct a copula given some partial information about the values that it assumes. One of the most common extension procedures is given by the multilinear interpolation (or checkerboard construction), which also plays a central role in characterizing dependence concepts for discrete random vectors. Here, the structure of the class of subcopulas is investigated and extension procedures (generalizing the multilinear interpolation) are presented in a high-dimensional framework to transform a specific subcopula to a copula. Moreover, convergence results are given in order to check how these extensions approximate (in different metrics) a target copula.

## Quantile-based risk sharing

*Paul Embrechts, ETH Zurich, Switzerland*

*Hailian Liu, University of Waterloo, Canada*

*Ruodo Wang, University of Waterloo, Canada*

We address the problem of risk sharing among agents using a two-parameter class of quantile-based risk measures, the so-called Range-Value-at-Risk (RVaR), as their preferences. The family of RVaR includes the Value-at-Risk (VaR) and the Expected Shortfall (ES), the two popular and competing regulatory risk measures, as special cases. We first present an inequality for RVaR aggregation, showing that a special form of subadditivity is satisfied by RVaR. Then, the risk sharing problem is solved by explicit construction. Three relevant issues in the optimal allocations are investigated: extra sources of randomness, comonotonicity, and model uncertainty. We show that in general, a robust optimal allocation exists if and only if none of the underlying risk measures is a VaR. Practical implications of the main results for risk management and policy makers are discussed, including gambling behaviour, moral hazard, regulatory arbitrage, and model misspecification. In particular, in the context of regulatory capital reduction, we provide some general guidelines on how a regulatory risk measure can lead to certain desirable or undesirable properties of risk sharing among firms, and show novel advantages of ES from the perspective of a regulator.

# Standardized Drought Indices: A novel uni- and multivariate approach

*Tobias Erhardt, Technische Universität München*

*Claudia Czado, Technische Universität München*

During past decades many different methods for drought quantification (drought indices) have been developed. However, most of them have crucial shortcomings and do not account for more than one or two variables which influence drought conditions and ignore inter-variable dependencies. We provide novel methodology for the calculation of (multivariate) drought indices. Inspired by existing approaches we compile a set of criteria which the novel, general, statistical drought indices should fulfill. Our novel approach is a stepwise implementation of these criteria, which omits the disadvantages of existing drought indices and provides a generalization to multivariate input. The multivariate indices are based on vine copulas, which are utilized to flexibly model multivariate non-Gaussian inter-variable dependence structures.

# Measures of Concordance for Copulas and Copulas for Measures of Concordance

*Sebastian Fuchs, Technische Universität Dresden*

We study a group of transformations mapping the collection of all copulas (of fixed but arbitrary dimension) into itself. These transformations may be used to construct new copulas from a given one or to prove that certain real functions on the unit cube are indeed copulas.

Applying this group, we then propose a concise definition of a measure of concordance for copulas. This definition, in which the axioms are formulated in terms of two particular subgroups of the group, provides an easy access to the investigation of invariance properties of a measure of concordance. In particular, it turns out that for copulas which are invariant under a certain subgroup the value of every measure of concordance is equal to zero.

In addition, we discuss a class of measures of concordance in which every element is defined as the expectation with respect to the probability measure induced by a fixed copula having an invariance property with respect to two subgroups of the group. This class is rich and includes the well-known examples Spearman's rho and Gini's gamma.

# A nonparametric copula estimator for the joint survival function of clustered right-censored event time data

*Candida Geerdens, Hasselt University*

*Paul Janssen, Hasselt University*

*Noel Veraverbeke, Hasselt University*

In survival analysis interest is in the time until a predefined event occurs (e.g., the time to death). Often, this event time is right-censored for some items in the study sample, i.e., only a lower time bound for the event is observed (e.g., by the end of the study death has not occurred). A further complexity can be grouping of the data (e.g., the study may include only twins, each twin then acts as a group of size two). Since clustered items share common traits, their event times show correlation.

Copulas provide a popular tool to describe the association in grouped time-to-event data. In a data setting where it is less evident to predetermine a parametric copula, such as the one of right-censored data, one may opt to apply a nonparametric copula. We define a new nonparametric copula estimator for the joint survival function of grouped right-censored event time data. In here, we consider two right-censoring schemes: univariate censoring and copula censoring. For the new nonparametric copula estimator, we establish the consistency and we assess the finite sample performance in various data settings via a simulation study. Focus is on the overall performance and the behavior in the extremal points of the unit square. A comparison with the recent nonparametric copula estimator of Gribkova and Lopez (2015) is given.

- [1] Geerdens, C., Janssen P. and Veraverbeke, N. (2015). Large sample properties of nonparametric copula estimators under bivariate censoring. *Statistics*, doi: 10.1080/02331888.2015.1119149.
- [2] Gribkova, S. and Lopez, O. (2015). Nonparametric copula estimation under bivariate censoring. *Scandinavian Journal of Statistics*, doi: 10.1111/sjos.12144.

## Modeling dependence in run-off triangles

*Christian Genest, McGill University, Montréal (Québec), Canada*

In order to determine appropriate reserves and risk capital for incurred but unpaid losses, property and casualty insurance companies must account for the fact that their lines of business may be dependent. For each line of business, payments relating to past claims are usually structured in a run-off triangle arranged to rows according

to the accident years, and to columns according to the development periods. Generalized linear models (GLMs) provide a convenient way to capture the influence of these two factors on the distribution of the loss ratios in each triangle. Following Shi & Frees (2011), copulas can then be used to combine these GLMs across lines of business.

To guard against the undesirable effects of an inadequate choice of dependence model on reserve estimation, a two-stage, rank-based inference procedure will be proposed to assist with copula selection and validation in this context. A hierarchical approach will also be advocated for the construction of flexible copulas. As emphasized by Arbenz et al. (2012), this modeling strategy relies on a conditional independence assumption whose implications will be highlighted. Under this assumption, the hierarchical structure can be constructed iteratively using rank-based clustering techniques, as detailed in Côté & Genest (2015). The entire approach will be illustrated with data for six lines of business from a large Canadian insurance company.

- [1] Arbenz, P., Hummel, C., Mainik, G., Copula based hierarchical risk aggregation through sample reordering, *Insurance: Mathematics and Economics* 51, 122-133, 2012.
- [2] Côté, M.-P., Genest, C., A copula-based risk aggregation model, *The Canadian Journal of Statistics* 43, 60-81, 2015.
- [3] Shi, P., Frees E.W., Dependent loss reserving using copulas. *ASTIN Bulletin* 41, 449-486, 2011.

## **Dependence modeling via conditional, partial and average copulas: inference and applications**

*Irène Gijbels, KU Leuven, Belgium*

In modeling dependencies via copulas the concepts of unconditional and conditional copulas have been widely studied in the literature. Since many association measures (Kendall's tau, Spearman's rho, ...) can be written as functionals of a copula, inference for these quantities follows from that of the copulas. In this talk we will also discuss the notions of partial and average conditional copulas and association measures. The emphasis in this talk is on semi-and nonparametric estimation of (un)conditional, average conditional and partial copulas and association measures. Conditional copulas couple conditional marginals to result into the conditional joint distribution function. A special situation occurs when the conditioning on a covariate(s) is such that it only influences the marginal distributions and not the copula. This simplifying assumption is often made in applications, but testing for it only received attention in recent

years. We briefly discuss inference under this setting and testing for the simplifying assumption to hold. We provide some illustrations of the various notions and methods and briefly illustrate their use in applications.

This talk is mainly based on joint work with Marek Omelka and Noël Veraverbeke.

## Risk-consistent conditional systemic risk measures

*Hannes Hoffmann, LMU*

We axiomatically introduce risk-consistent conditional systemic risk measures defined on multidimensional risks. This class consists of those conditional systemic risk measures  $\rho$  which can be decomposed into a state-wise conditional aggregation  $\Lambda$  and a univariate conditional risk measure  $\eta$ . Our studies extend known results for unconditional risk measures on finite state spaces. In the course of this axiomatic description we identify the relationship between the properties of  $\rho$  and the properties of  $\eta$  and  $\Lambda$ , and vice versa. Our approach covers many prominent examples of systemic risk measures from the literature and used in practice like the conditional Value at Risk from [2] or the systemic expected shortfall from [1]. Finally, we present a small simulation study on the Value at Risk with an aggregation function which allows to incorporate direct contagion effects within the financial system as well as the injection of capital from an external participant, e.g. a regulator.

This is a joint work with T. Meyer-Brandis and G. Svindland.

- [1] V. Acharya, L. Pedersen, T. Philippon, M. Richardson, Measuring systemic risk, 2010.
- [2] T. Adrian, M.K. Brunnermeier, CoVaR, Technical Report, National Bureau of Economic Research, 2011.

## Modeling dependent credit spreads using random fields

*Amelie Hüttner, Technische Universität München,  
Matthias Scherer, Technische Universität München,  
Benedikt Gräler, Ruhr-Universität Bochum*

The joint dynamics of CDS spreads, written on European corporate bonds as reference entity, are modeled and estimated. Temporal dependence is captured via classical time series models. Their multivariate innovations –this is the main innovation of our study– are described using random fields, for which we adapt methods from geostatistics. We emphasize the advantages of this methodology with respect to the

flexibility of the model in its dimension, a convenient parameterisation of the dependence structure, and the possibility to estimate it in situations with partially missing data. Finally, we describe our empirical findings and compare the results obtained in credit market to stock markets.

## **Fat tails and copulas: Limits of diversification revisited**

*Rustam Ibragimov, Imperial College Business School*

*Jingyuan Mo, NYU Stern School of Business*

*Artem Prokhorov, University of Sydney Business School*

We consider the problem of portfolio risk diversification in a Value-at-Risk framework with heavy-tailed risks and arbitrary dependence captured by a copula function. We use the power law for modelling the tails of risk distributions and investigate whether the benefits of diversification persist when the risks in consideration are allowed to have extremely heavy tails with tail indices less than one and when their copula describes wide classes of dependence structures. We show that for asymptotically large losses with the Eyrraud-Farlie-Gumbel-Morgenstern copula, the threshold value of tail indices at which diversification stops being beneficial is the same as for independent losses. We further extend this result to a wider range of dependence structures which can be approximated using power-type copulas and their approximations. This range of dependence structures includes many well known copula families, among which there are comprehensive, Archimedian, asymmetric and tail dependent copulas. In other words, diversification increases Value-at-Risk for tail indices less than one regardless of the nature of dependence between portfolio components within these classes.

## **Multivariate Stochastic Characterisation of Sea States**

*W.S. Jäger, Delft University of Technology*

*O. Morales Napoles, Delft University of Technology*

A large part of the Netherlands is below sea level. At the same time this area is densely populated and contains a significant share of the country's economic value, which will be exposed in case of flooding. To understand the flood risk, it is important to have realistic estimates of the probability of occurrence of different flood scenarios. A main threat are coastal storms with severe surges. While oceanographic variables have been recorded for many decades along the Dutch coast, the number



of observed extreme events is still low. A storm event creating a ten thousand year return flood, which Dutch dikes are designed to withstand, has never been recorded. This research explores a vine-copula approach to estimate the joint distribution of oceanographic variables (water levels, wave heights, etc.). Using pair-copulas with upper tail dependence and asymmetry properties, we can find a distribution based on the available data records with mostly normal events and a small number of extreme events. The approach can be used to develop synthetic storm scenarios of different return periods. These are important to predict potential levee failures and flood impacts as well as to determine design criteria and to conduct reliability analysis of flood defence structures.

## **Are the solutions of Copula ARMA-GARCH models exponentially ergodic?**

*Piotr Jaworski, University of Warsaw*

Copula ARMA-GARCH models are among the most popular models of multivariate financial returns. The reason is that they allow two stage estimation. First ARMA-GARCH estimation of margins, then the copula based estimation of dependence between innovations. In my talk I am going to discuss two issues.

1. Which Copula ARMA-GARCH models allow the stationary and ergodic solutions?
2. How fast do the other solutions converge to stationary and ergodic ones?

The positive answer for both questions has important practical consequences. Namely the exponential rate of convergence of an arbitrary solution to the stationary ergodic one is crucial in the study of the asymptotic properties of maximum likelihood estimators.

[1] Daniel Straumann, Estimation in Conditionally Heteroscedastic Time Series Models, Springer 2005.

## **Copula models combining groups of variables**

*Harry Joe, University of British Columbia, Canada*

When there are a large number of variables, and the variables can be divided into groups, one can consider copula models that use the group information. Several methods based on various conditional independence assumptions can be used to construct such models. Computing techniques for estimation will be presented and applications will be mentioned.

# Model distances for vine copulas in high dimensions

*Matthias Killiches, Technische Universität München*

*Daniel Kraus, Technische Universität München*

*Claudia Czado, Technische Universität München*

Vine copulas are a flexible class of dependence models consisting of bivariate building blocks and have proven to be particularly useful in high dimensions. Classical model distance measures require multivariate integration and thus suffer from the curse of dimensionality. In this paper we provide numerically tractable methods to measure the distance between two vine copulas even in high dimensions. For this purpose, we consecutively develop three new distance measures based on the Kullback-Leibler distance, using the result that it can be expressed as the sum over expectations of KL distances between univariate conditional densities, which can be easily obtained for vine copulas. To reduce numerical calculations we approximate these expectations on adequately designed grids, outperforming Monte Carlo-integration with respect to computational time. In numerous examples and applications we illustrate the strengths and weaknesses of the developed distance measures.

- [1] Killiches, M., Kraus, D. and Czado, C. (2015) Model distances for vine copulas in high dimensions with application to testing the simplifying assumption (submitted for publication)

## D-vine Copula Based Quantile Regression

*Daniel Kraus, Technische Universität München*

*Claudia Czado, Technische Universität München*

Quantile regression, that is the prediction of a random variable's quantiles conditioned on other random variables taking on certain values, has steadily gained importance in statistical modeling and financial applications. We introduce a new semi-parametric quantile regression method based on sequentially fitting a likelihood optimal D-vine copula to given data resulting in highly flexible models with easily extractable conditional quantiles. As a subclass of regular vine copulas, D-vines enable the modeling of multivariate copulas in terms of bivariate building blocks, a so-called pair-copula construction (PCC). The proposed algorithm works fast and accurate even in high dimensions and incorporates an automatic variable selection by maximizing the conditional log-likelihood. Further, typical issues of quantile regression such as quantile crossing or transformations, interactions and collinearity of variables are automatically taken care of. In a simulation study the improved accu-

racy and saved computational time of the approach in comparison with established quantile regression methods is highlighted. An extensive financial application to international credit default swap (CDS) data including stress testing and Value-at-Risk (VaR) prediction demonstrates the usefulness of the proposed method.

## Mixed discrete and continuous modeling with vine copula

*Dorota Kurowicka, Delft University of Technology*

*Aurelius Zilko, Delft University of Technology*

We present the application of vine copula approach to a problem containing mixed discrete and continuous variables. Four different models are constructed for the data collected for the purpose of predicting the length of disruption caused by problems with the train detection system in the Dutch railway network and their performance is tested. The vine copula model where some conditional copulas are allowed to directly depend on the conditioning variables represents the data very well.

## Testing the simplifying assumption in high-dimensional vine copulas

*Malte S. Kurz, Ludwig-Maximilians-Universität München*

*Fabian Spanhel, Ludwig-Maximilians-Universität München*

In almost all applications of vine copulas, it is assumed that the data generating process satisfies the simplifying assumption, i.e., every conditional copula in a vine collapses to an unconditional copula. So far, tests for the simplifying assumption have been limited to three-dimensional vine copulas. Testing the simplifying assumption in high-dimensional vine copulas is a difficult task, because tests are based on estimated data and amount to checking constraints on high-dimensional distributions. We propose a novel testing procedure which utilizes decision trees to mitigate the curse of dimensionality by searching the possibly strongest deviation from the simplifying assumption. The proposed test is computationally feasible for high-dimensional (e.g.,  $d = 50$ ) data sets. Its finite sample performance is analyzed in an extensive simulation study. To demonstrate the applicability of our test, we investigate whether the simplifying assumption is reasonable in practical applications.

# Value-at-Risk Bounds with Partial Dependence Information

*Thibaut Lux, Technical University Berlin*

*Antonis Papantoleon, Technical University Berlin*

We study the problem of finding bounds on Value-at-Risk (VaR) in the presence of model uncertainty. In particular we consider a portfolio, consisting of  $d$  risks  $X_1, \dots, X_d$  whose individual distributions are known whereas only partial information on the dependence structure of the constituents is available. This is in the literature often referred to as (partial) dependence uncertainty. In this context, a large part of the academic research has focused on the computation or estimation of bounds on the VaR of the sum  $\sum_{i=1}^d X_i$  in the absence of information on the dependence structure. These bounds however tend to be impractically wide as they omit all information on the dependence between the risks. Therefore, methods to estimate bounds on the aggregate VaR which account for additional information about the interaction between the individual risks are becoming increasingly popular. We account for two types of additional dependence information in the computation of VaR bounds. Firstly, we obtain bounds in the case in which the joint law of  $(X_1, \dots, X_d)$  is known on a compact subset of  $\mathbb{R}^d$ . This generalizes the setting in Bernard and Vanduffel (2015) such as to integrate information on the lower-dimensional marginals of the joint distribution. Secondly, we consider information about the distance between the joint distribution of  $(X_1, \dots, X_d)$  and a given reference distribution with respect to a metric on the set of distributions. The solutions are based on an improvement of the Fréchet-Hoeffding bounds which account for the partial information on the dependence structure. By an extension of the results of Embrechts and Puccetti (2006) we are able to translate the improved Fréchet-Hoeffding bounds to estimates for the maximal and minimal VaR. Furthermore, we provide a benchmark analysis comparing the extended rearrangement algorithm presented by Bernard and Vanduffel (2015) to our approach.

- [1] Bernard, C, Vanduffel, S, A new approach to assessing model risk in high dimensions, *Journal of Banking and Finance* 58, 166-178, 2015.
- [2] Embrechts, P, Puccetti, G, Bounds for functions of dependent risks, *Finance and Stochastics* 10, 341-352, 2006.

## Exchangeable exogenous shock models

*Jan-Frederik Mai, XAIA Investment*

*Steffen Schenk, Technische Universität München*

*Matthias Scherer, Technische Universität München*

In [1], we study the family of  $d$ -dimensional copulas which is obtained by ordering, distorting, and multiplying its input arguments. An analytical characterization is achieved, and a stochastic interpretation is found that links this family of multivariate distribution functions to exogenous shock models. Moreover, an alternative, de Finetti-type stochastic model based on increasing additive processes is established. The considered family of copulas comprises some interesting subfamilies. The most prominent subfamily are exchangeable Marshall-Olkin copulas, which are well-studied in the literature, see, e.g., [4, 5]. One new subfamily based on Sato processes gives rise to a copula-characterization of self-decomposable probability laws on the positive real axis, see [2]. Another subfamily is based on the concept of a Dirichlet prior from Bayesian statistics and is well-suited for some applications in risk management, see [3].

- [1] Mai J.-F, Schenk S, Scherer M, Exchangeable exogenous shock models, *Bernoulli* 22, 1278-1299, 2016.
- [2] Mai J.-F, Schenk S, Scherer M, Two novel characterizations of self-decomposability on the half-line, *Journal of Theoretical Probability*, forthcoming, 2016.
- [3] Mai J.-F, Schenk S, Scherer M, Analyzing model robustness via a distortion of the stochastic root: a Dirichlet prior approach, Working paper, 2016.
- [4] Mai J.-F, Scherer M, Lévy-frailty copulas, *Journal of Multivariate Analysis* 100, 1567-1585, 2009.
- [5] Mai J.-F, Scherer M, Reparameterizing Marshall-Olkin copulas with applications to sampling, *Journal of Statistical Computation and Simulation* 81, 59-78, 2011.

## Flood frequency analysis by using copulas

*Luis Mediero, Technical University of Madrid, Spain*

*Ana I. Requena, INRS-ETE*

Flood frequency analyses are usually univariate and only account for the peak discharge of the hydrograph, as in most cases there is a direct relationship between the river discharge and water level at the point of interest. However, in some cases this direct relationship does not exist, such as the cases of floodplains and dams. Consequently, a multivariate analysis is required to characterise hydrographs by using a set of hydrograph variables, such as peak discharge, hydrograph volume and hydrograph duration. Copulas have shown themselves as being suitable when characterising dependence between such hydrograph variables. Copulas have been applied to solve

several problems encountered in flood frequency analysis. Univariate regionalisation techniques are extended to the bivariate case either to improve the at-site estimates of flood peak and hydrograph volume quantiles for a given return period at gauged locations or provide estimates at ungauged sites. They are also used to characterise the dependence between peak discharge and hydrograph volume to establish the design water levels at dams, as well as to assess the dam safety against floods. Bivariate flood trends are studied by analysing how copula dependence varies in time. Finally, an uncertainty analysis to find the minimum number of pairs required to obtain reliable flood quantile estimates is conducted. A set of case studies in Spain and the United Kingdom are used to provide an insight into how copulas have improved the traditional techniques used in flood frequency analysis.

## Representing sparse Gaussian DAGs as sparse R-vines allowing for non-Gaussian dependence

*Dominik Müller, Technische Universität München*

*Claudia Czado, Technische Universität München*

Modeling dependence in high dimensional systems has become an increasingly important topic. Most approaches rely on the assumption of a joint Gaussian distribution such as statistical models on directed acyclic graphs (DAGs). They are based on modeling conditional independencies and are scalable to high dimensions. In contrast, vine copula based models can accommodate more elaborate features like tail dependence and asymmetry. This flexibility comes however at the cost of exponentially increasing complexity for model selection and estimation. We show a connection between these two model classes by giving a novel representation of DAG models in terms of sparse vine models. Therefore we can exploit the fast model selection and estimation of sparse DAGs while allowing for non-Gaussian dependence in the vine models. We demonstrate for a high dimensional data set that this approach outperforms standard methods for vine structure estimation.

[1] Müller, D., Czado, C., Representing sparse Gaussian DAGs as sparse R-vines allowing for non-Gaussian dependence (submitted for publication).

## Generalized Additive Models for Pair-Copula Constructions

*Thomas Nagler, Technische Universität München*

Pair-copula constructions (PCCs) are flexible models for the dependence in a random vector and attracted a lot of interest in recent years. In this paper, We use generalized

additive models to extend PCCs to allow for effects of covariates on the dependence parameters. We let each pair-copula parameters depend directly on the covariates in a parametric, semi-parametric or non-parametric way. We propose a sequential estimation method that we study with simulations. Finally, we apply our method to investigate the time-varying dependence structure between the intraday returns on five major foreign exchange rates.

## **Modeling the structure of multivariate non-normal longitudinal data by pair copula decomposition**

*Marta Nai Ruscone, LIUC and Università Cattolica del Sacro Cuore*

The presence of longitudinal data is increasing in the scientific areas. In longitudinal data several variables are measured over a sample of statistical units at different times. Therefore longitudinal data are characterized by both between variables dependence and across time dependence. Here, we propose a new flexible approach of modeling these dependencies between the components of non-normal multivariate longitudinal data by using copulae.

The proposed approach considers two levels. The first level models the dependencies between any group of variables at each point in time using copula. This results in a longitudinal series of dependencies for each group of variables. The second level models each of these longitudinal series using a pair copula decomposition to describe the dependence between the variables across time.

The use of the pair copula decomposition allows to overcome the problem of the multivariate copulae which lacks flexibility to model dependence and have other limitations, such as parameter restrictions.

We provide a comprehensive study of complex and hidden dependence patterns in longitudinal non-normal data. Furthermore, we illustrate the applicability and the advantages of the proposed approach using simulated and real data.

- [1] Smith M., Min A., Almeida C., Czado C., Modeling longitudinal data using a pair-copula decomposition of serial dependence, *Journal of the American Statistical Association* 105, 2010.
- [2] Lambert P., Vandenhende F., A copula-based model for multivariate non-normal longitudinal data: analysis of a dose titration safety study on a new antidepressant, *Statistics in medicine* 21, 3197-3217, 20102.

## **Estimating extremal dependence using B-splines**

*Johanna G. Nešlehová, McGill University, Montréal (QC), Canada*

B-spline smoothing techniques are commonly used in functional data analysis. In this talk, I will explain how this tool can be adapted to derive intrinsic estimators of the Pickands dependence function characterizing the dependence in the maximum attractor of a bivariate continuous distribution. The approach is rooted in a rank-based transformation of the data due to Cormier et al. (2014). As shown therein, a plot of the transformed pairs of points provides a useful tool for detecting extreme-value dependence or extremal tail behavior. When the case arises, a constrained B-spline can be fitted through a suitable subset of the points to get an estimator of the Pickands dependence function associated with the extreme-value attractor. This estimator is intrinsic, i.e., it satisfies all the conditions required to qualify as a Pickands dependence function. The excellent finite-sample performance of this estimator was documented through simulations by Cormier et al. (ibid.).

As a follow-up to this work, I will state minimal conditions under which this estimator is consistent, and I will give its limiting distribution. This result is valid whatever the order of the B-splines. I will also demonstrate through theory and simulations that while B-splines of order 3 are sufficient to estimate the Pickands dependence function with accuracy, B-splines of order 4 are essential to grasp the features of the spectral distribution associated with the maximal attractor. This approach leads to an estimator that generally outperforms the maximum empirical likelihood estimator studied by Einmahl & Segers (2009). This talk is based on joint work with A. Bücher, C. Genest, and D. Sznajder.

- [1] Cormier, E., Genest, C., Nešlehová, J.G., Using B-splines for nonparametric inference on bivariate extreme-value copulas, *Extremes* 17, 633-659, 2014.
- [2] Einmahl, J.H., Segers, J., Maximum empirical likelihood estimation of the spectral measure of an extreme-value distribution, *The Annals of Statistics* 37, 2953-2989, 2009.

## Copula mixed effect models for multivariate meta-analysis

*Aristidis K. Nikoloulopoulos, University of East Anglia, UK*

Synthesis of diagnostic test accuracy studies is the most common medical application of multivariate meta-analysis [1]. A generalized linear mixed model (GLMM) is currently recommended to synthesize diagnostic test accuracy studies. We propose copula mixed models for multivariate meta-analysis in this context [2, 3, 4]. Our general models include the GLMM as a special case and can also operate on the original scale of sensitivity and specificity. Summary receiver operating characteristic curves are deduced for the proposed model through quantile regression techniques



and different characterizations of the bivariate random effects distribution. Our general methodology is demonstrated with extensive simulation studies and illustrated by re-analysing the data of several published meta-analyses. Our study suggests that there can be an improvement on GLMM in fit to data and makes the argument for moving to copula random effects models especially because of their richness including reflection asymmetric tail dependence, and, computational feasibility.

- [1] Jackson, D., Riley, R., White I.R., Multivariate meta-analysis: Potential and promise. *Statistics in Medicine* 30, 2481-2498, 2011.
- [2] Nikoloulopoulos, A.K., A mixed effect model for bivariate meta-analysis of diagnostic test accuracy studies using a copula representation of the random effects distribution, *Statistics in Medicine* 34, 3842-3865, 2015.
- [3] Nikoloulopoulos, A.K., A vine copula mixed effect model for trivariate meta-analysis of diagnostic test accuracy studies accounting for disease prevalence, *Statistical Methods in Medical Research*, <http://dx.doi.org/10.1177/0962280215596769>DOI:10.1177/0962280215596769.
- [4] Nikoloulopoulos A.K., CopulaREMADA: Copula mixed effect models for bivariate and trivariate meta-analysis of diagnostic test accuracy studies 2015. <http://CRAN.R-project.org/package=CopulaREMADA><http://CRAN.R-project.org/package=CopulaREMADA>, R package version 0.9.

## Hierarchical Realized Copula with Application to VaR

*O. Okhrin, Technische Universität Dresden*

*A. Teterova, Universität St. Gallen*

A computationally simple estimator for multivariate hierarchical Archimedean copulae (HAC) is introduced. It is proposed to estimate the structure and the parameters of a copula based on the correlation matrix only. The advantage of this estimator is the significant reduction of the computational costs and that it can be used in cases when the maximum likelihood (ML) type estimation can not be performed. Extensive simulation studies show the performance of the proposed estimator in comparison to the benchmark models. The application of the estimator to the one-day ahead Value at Risk (VaR) prediction using high-frequency data gives rise to the hierarchical realized copula which exhibits good forecasting properties for a multivariate portfolio in comparison to the competitor models.

# New copulas based on general partitions-of-unity and their applications to risk management

*Dietmar Pfeifer, Universität Oldenburg*

*Hervé Aoumlac Tsatedem*

*Andreas Maendle*

*Côme Girschig*

We construct new multivariate copulas on the basis of a generalized countably infinite partition-of-unity approach. This approach allows - in contrast to finite partition-of-unity copulas - for tail-dependence as well as for asymmetry. A possibility of fitting such copulas to real data from quantitative risk management is also pointed out.

- [1] C. Cottin and D. Pfeifer: From Bernstein polynomials to Bernstein copulas. *Journal of Applied Functional Analysis* (2014), 277-288.
- [2] D. Pfeifer and D. Lauterbach: Some Extensions of Singular Mixture Copulas. In: M. Halin, D. Mason, D. Pfeifer, J. Steinebach (Eds.): *Mathematical Statistics and Limit Theorems - Festschrift in Honour of Paul Deheuvels*, Springer (2015), Heidelberg, 271 - 286.
- [3] D. Pfeifer, H.A. Tsatedem, A. Maendle and C. Girschig: New copulas based on general partitions-of-unity and their applications to risk management. Preprint (2015), arXiv:1505.00288v2

# Ambiguity with respect to copulas and portfolio optimization

*Mathias Pohl, University of Vienna*

*Georg Pflug, University of Vienna*

Model ambiguity refers to the situation in which not only the realizations of random variables is uncertain but also their probability model/law is unknown. Significant knowledge gaps exist concerning the ambiguity in the choice of copula models. To fill these gaps, the well-known Wasserstein distance is introduced to the theory of copulas in order to measure the distance between two copulas. New theoretical results linking the optimal transportation between copulas and the Spearman's correlations coefficients are established. This indicates that the Wasserstein distance is a meaningful measure for the distance between copulas. Therefore, it is reasonable to define an ambiguity set of copulas as the set of copulas which are close to a pre-specified copula with respect to the Wasserstein distance.

The second part of the talk focuses on an application of this concept to the portfolio selection problem. Similar to [1], we consider the portfolio optimization problem with respect to the average value at risk (also known as expected shortfall), which can be computed using copulas. We present a framework which is robust against a possible misspecification of the copula as we suggest a minimax problem which relies on the ambiguity set of copulas.

[1] Kakouris, I., Rustem, B., Robust portfolio optimization with copulas, *European Journal of Operational Research* 235(1), 28 - 37, 2014.

## **The minimum of the expected value of the product of three uniform random variables: a problem (finally) solved**

*Giovanni Puccetti, University of Milan, Italy*

We illustrate the history and two different solution methods of this problem which has been open for more than 30 years. These methods have opened new research fields in applied probability, statistics, financial mathematics and actuarial science.

## **Nonparametric estimation of counterfactual distributions for Munich rent survey**

*C. Schellhase, Bielefeld University, Germany*

*J. Schnurbus, University of Passau, Germany*

*G. Kauermann, University of Passau, Germany*

Recently it is discussed, whether the rent increase for German major cities is merely a matter of rising demand that is exploited by flat owners. We provide a counterfactual distribution-based decomposition analysis of several current releases of the Munich rent survey that allows to disentangle the rent increase over time into two effects. First, the rent increase caused by an improvement of the flats and second, the increase in terms of equivalent flats simply getting more expensive.

Two new nonparametric estimators for the counterfactual distribution by simultaneous smoothing of continuous and discrete covariates are proposed: (i) A penalized B-splines estimator for mixed-data pair-copulas in vine structures and (ii) a mixed kernel estimator. The properties of both estimators are shown in a Monte Carlo simulation study. Both estimators yield that for Munich the majority of the rent increase is not justified by improved flat characteristics.

# Vine copulas in synthetic flood events generation

*Radek Solnický, Aon Benfield Impact Forecasting*

Probabilistic catastrophe models are tools used by (re)insurance companies to evaluate loss potential to their portfolio caused by natural perils. Due to the complexity of the flood events and the changes of insured portfolios from year to year this potential cannot be calculated by simple statistical models using the historical observed losses. Therefore models based on modelling of the underlying physical phenomenon are used for evaluation of flooding on current insured portfolio. Moreover, as the probabilistic results are required, it is necessary to model a whole set of flood scenarios which are likely to occur. Design of such synthetic events catalogue is an essential part of the probabilistic catastrophe model development. Apart from the flood event occurrence frequency, the river discharges in affected areas have to be determined for each event from the catalogue. While being consistent with the observed data at gauging stations these events should also provide realistic spatial patterns. This represents a high dimensional task which can be addressed by use of the vine copulas methodology with the structure of the vine influenced by the river network. With the help of the VineCopula R package a wide range of copulas can be utilized.

- [1] Graeler, B., van den Berg, M., Vandenberghe, S., Petroselli, A., Grimaldi, S., Baets, B. D., Verhoest, N., 2013. Multivariate return periods in hydrology: a critical and practical review focusing on synthetic design hydrograph estimation. *Hydrology and Earth System Sciences* 17 (4), 12811296.
- [2] Salvadori, G., De Michele, C., Kottegoda, N. T., Rosso, R., 2007. *Extremes in nature: an approach using copulas*. Vol. 56. Springer Science & Business Media.
- [3] Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., 2015. *VineCopula: Statistical Inference of Vine Copulas*. R package version 1.6.
- [4] Aas, K., Czado, A., Frigessi, H., Bakken (2009): Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics*, 44, pp.182 - 198.
- [5] Bedford, T. and R.M. Cooke (2002): Vines - a new graphical model for dependent random variables. *Annals of Statistics*, 30 (4), pp. 1031 - 1068.
- [6] Joe, H. (1996): Families of m-variate distributions with given margins and  $m(m-1)/2$  bivariate dependence parameters. In L. Rüschendorf and B. Schweizer and M. D. Taylor (Ed.), *Distributions with Fixed Marginals and Related Topics*.

# A dynamic regime switching copula approach for dependence modeling of the S&P500 and the WTI oil prices

*Manel Soury, AMSE Greqams*

In this paper we propose a dynamic copula model where the parameter of the dependence depends on a hidden first order markov chain: the regime switching copula (RS copula). We analyse the dependence and comovements between SP500 index (as an indicator of the economic activity in the US) and the WTI crude oil (as an indicator of the energy market ) in terms of their conditional volatility. We also consider the results given by the time varying copula model without regime switching based on Patton (2006). The marginals of the returns are modeled by the GARCH type and the GAS model, to extract the standardized residuals from each return and then uniform inputs are obtained using the empirical cumulative distribution function for the residuals. Both the dynamic and the RS copulas are then fitted to the new transformed data and used to measure the time evolution of their dependence. Empirical results showed that first the dynamic nature of the dependence structure between both returns, it increased considerably around the financial crisis, and alternated between negative and small values before, second the existence of two persistent regimes (high and low) concerning the dependence was proven by the dynamic RS copula model and third we confirm the asymmetry between both returns from the fact that the tail dependence is more important when in the high regime than in the low dependence one, or it can be inexistent, and we confirm the fact that dependence in the lower tails are indeed higher than the upper one.

## Generalized Ideal Parent (GIP): Discovering non-Gaussian Hidden Variables

*Yaniv Tenzer, Department of statistics, the Hebrew University*

*Gal Elidan, School of computer science, the Hebrew University*

*Ilya Slovetichik, School of computer science, the Hebrew University*

*Ami Wiesel, School of computer science, the Hebrew University*

A formidable challenge in uncertainty modeling in general, and when learning Bayesian networks in particular, is the discovery of unknown hidden variables. Few works that tackle this task are typically limited to discrete or Gaussian domains, or to tree structures. We propose a novel general purpose approach for discovering hidden variables in flexible non-Gaussian domains using the powerful class of Gaussian copula networks. Briefly, we define the concept of a hypothetically optimal predictor of

variable, and show how it can be used to discover useful hidden variables in the expressive framework of copula networks. Our approach leads to performance and compactness advantages over competitors in a variety of domains.

## Risk bounds for factor models

*Steven Vanduffel, Vrije Universiteit Brussel*

*Carole Bernard, Grenoble Ecole de Management*

*L. R. Rüschendorf, University of Freiburg*

*Ruodu Wang, University of Waterloo*

Recent literature has been investigating the risk aggregation of a portfolio  $X = (X_i)_{1 \leq i \leq n}$  under the sole assumption that the marginal distributions of the risks  $X_i$  are specified but not their dependence structure. Clearly, there exists a range of possible values for the risk measure of  $S = \sum_{i=1}^n X_i$  and it has been found that the dependence uncertainty spread, as measured by the difference between the upper bound and the lower bound on these values, is typically too wide to be useful in practice. Consequently, it is necessary to consider additional information on the dependence among the risks. Here, we study a partially specified factor model in which each risk  $X_i$  has a known joint distribution with the common risk factor  $Z$ , but we dismiss with the conditional independence assumption that is typically made in fully specified factor models. We derive easy to compute bounds on risk measures such as Value-at-Risk (VaR) and law-invariant convex risk measures (e.g., Tail Value-at-Risk (TVaR)) and demonstrate their (asymptotic) sharpness. We show that the dependence uncertainty spread is typically substantially reduced and that contrary to the case in which only marginal information is used it is *not* necessarily larger for VaR than for TVaR.

## Valuation of Multi-Name Credit Derivatives with Factor Copulas

*Thibault Vatter, University of Lausanne*

*Damien Akerer, EPFL and Swiss Finance Institute*

This paper describes a flexible and tractable framework to model portfolio losses and price multi-name credit derivatives. Assuming losses have a discrete support we recover the exact portfolio loss density and the exact CDO-squared loss density using discrete Fourier inversion. We provide the canonical construction of factor models for default times by expressing the dependence structure in terms of conditional copulas. This representation enables one to arbitrarily specify the sensitivity of each individual default time to the systemic factors. We introduce the Beta-binomial model to account for stochastic recovery rates correlated with default times. Finally,

we illustrate the flexibility and computational efficiency of the setup with relevant numerical examples and calibrations to index tranches.

## **Sum of two uniform random variables**

*Ruodo Wang, University of Waterloo, Canada*

We study possible distributions of the sum of two dependent  $U[-1, 1]$  random variables. A full characterization is not yet found.

# Posters

## Determination of Nonlinear Dependency on Solvency II Requirements using Copulas

*Etkin Hasgül, Middle East Technical University, Ankara-Turkey*

*Sevtap Kestel, Middle East Technical University, Ankara-Turkey*

Solvency II directive proposes that the economic capital which is determined by using Solvency Capital Requirements (SCR) is needed to have a probability of ruin limited to 0.5% whose evaluation is based on VaR and CVaR as volatility measures [1]. A common SCR for a combination of different branches is calculated by not only aggregation of these branches but also considering possible dependencies. Thus, dependence between different risk factors related to corresponding branches should be known. If the association is not linear or cannot be transformed into a linear shape, using linear methods such as Pearson correlation ( $\rho$ ) and Kendal's Tau ( $\tau$ ) will cause an incompatible calculation and a gap between the allocated reserve and the actual requisite reserve. Therefore, the dependence models which can also represent the non-linear relationships should be used in the calculation of common SCR. Consequently, more precise measurements on the risk dependencies make insurer take more secured decisions regarding the efficient allocation of the funding for (re-)insurance portfolios [2].

The study aims to test how effective and realistic both elliptical and archimedean copulas over dependency in common SCR are and to determine dependence values in Solvency II criteria for emerging markets such as Turkey. For this purpose, non-life insurance companies in Turkey are taken into account to determine the correlation structure among the lines of business via traditional correlation coefficient measures and the copula families. A comparison on their efficiencies is done through sensitivity analyses.

- [1] Nguyen T. et al. Risk Aggregation by Using Copulas in Internal Models, Journal of Mathematical Finance, 2011(1), 50-57, 2011.
- [2] Zlateva P., Velev D. Measuring Risk Dependencies Due to Two Natural Disasters by Bivariate Copula, International Proceedings of Economics Development & Research, 2012(41), p182, 2012.



# **An Anderson-Darling approach for assessing the multivariate goodness of fit**

*Andreas Mändle, Universität Oldenburg*

When modeling risk, e.g. in fields like finance and insurance, we often face the problem of modelling extremal events. This involves making distributional assumptions for (sometimes multivariate) data. It has been frequently observed that in practice tails are heavier than normal and extremes appear in clusters, indicating tail dependence. In such cases the assumptions of normality are violated. Therefore there is often uncertainty if the normal assumption can still be justified. In the univariate case a popular method of testing the assumption of normality is by using the Anderson-Darling test. It is known for its strong power, especially when there are deviations in the tails of a distribution. We will consider a possible generalization of this test to the multivariate case. Its application in a multivariate test seems inconvenient, as the calculation of the n-variate test statistic required the calculation of an n-dimensional integral. To facilitate this issue a calculation formula of this multivariate Anderson-Darling statistic for finite, multidimensional samples will be presented. Using this formula immensely simplifies the calculation and thus serves as one key ingredient for the practical use of the test. As a practical example, the test has been applied in an example from hydrology where the test is used to assess the goodness of fit of four different model approaches to capture time dependence and extreme value behaviour of river discharge data.

## **Method to choose a Archimedean copula optimal**

*Diana C. Moreno, Universidad Nacional de Colombia*

*Liliana Blanco, Universidad Nacional de Colombia*

*Wilmer Pineda, Universidad Santo Tomas, Colombia*

Nowdays, copula are every strong tool model data in which dependence among random variable is important and there is an absence of the normality. However, one of the highest changes for a researches is to select the adequate copula since there isn't any standar method to do so. A methodology to select the best archimedean copula is intended to be reached by comparing the methods of nonparametric, parametric and semiparametric estimation taking into account graphic and analytic methods. It's intended to select the copula that adjust the best to the data water level and Concentration corresponding to the Nemizaque station on the Fonce river located in Santander-Colombia. For this, we use the package copula, copBasic and nacopula of R statistical software.

- [1] Haff, I. H. & Segers, J. (2015), Nonparametric estimation of pair copula constructions with the empirical pair copula, *Computational Statistics & Data Analysis* 84(0), 1-13.
- [2] Yang, D. (2010). Copula and Default Correlation. Master thesis, Louisiana State University and Agricultural and Mechanical College.
- [3] Durante, F. and C. Sempi (2010). Copula Theory: an Introduction. In Workshop on Copula Theory and its Applications, Volume 198, Berlin-Heidelberg,, pp. 3-31.
- [4] Genest, C. and J.-C. Boies (2003). Detecting dependence with kendall plots. *The American Statistician* 57 (4), 275-284.

## Assessment of Multivariate Drought Index via Vine Copula

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Drought is one of the common natural hazards, spreading across large areas and affecting a great population concurrently, mainly having different types like meteorological, agricultural, hydrological, and socioeconomical [1]. In recent years, most of the drought risk mitigation mechanisms are generated based on different indices. Naturally, considering many drought indicators and their associations simultaneously provide more flexible drought mitigation [2]. For this reason, vine copula approach comes into play to generate multivariate drought index, by combining a set of indices.

In this study, vine copula based multivariate drought index is presented to combine different drought indices by incorporating the variable dependencies. As an application, the proposed method is applied to identify drought conditions of Kulu station in Konya, Turkey. Initially, standardized drought indices are generated using the set of drought relevant parameters such as precipitation, temperature, evapotranspiration etc. Afterwards, constructed indices with a given time scales are flexibly linked together via vine copula methodology. The multivariate vine copula based drought index is compared with the existed different drought indices. Besides, standardized drought index based duration and severity are defined and embedded into vine copula modeling in order to capture the association between generated indices and their severity and duration properties.

- [1] Heim, R. R., A review of 20th-century drought indices used in the United States, Bull Am. Meteorol Soc., 83(8), 1149-1166, 2002.
- [2] Erhardt, T. M. and Czado, C., Standardized drought indices: A novel uni- and multivariate approach (submitted), arXiv:1508.06476 [stat.AP], 2015.

## **Improved Frechet-Hoeffding bounds on $d$ -copulas and applications in model-free finance**

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We consider the problem of finding model-free bounds for prices of multi-asset options (i.e. options on multiple underlyings) in the case when partial information of the assets probability distribution is available. We focus on the case in which the one-dimensional marginal distribution of each individual asset is known while also partial information on the dependence structure between the assets is available. This is in the literature often referred to as dependence uncertainty. The problem has been extensively studied in the two-asset case for which solutions were given by Tankov (2011) and Bernard et al. (2012). We extend these results to options that depend on more than two assets. The solution is based on an improvement of the classical Frechet-Hoeffding bounds that allows for a representation of partial information on the dependence structure. By an extension of the results of Mueller and Stoyan (2003) on multivariate stochastic dominance we are able to show that the improved bounds can be interpreted as minimal or maximal distributions with respect to the lower orthant order. The link between the lower orthant order on the set of distribution functions and the prices of multi-asset options is established via a multivariate partial integration formula. Numerical results show that the improved bounds are indeed narrower than the 'classical' ones.

## **A Vine Copula-GARCH Approach to Corporate Portfolio Risk**

*Christopher Wells, Chatham Financial,*

*Maciej Rys, Chatham Financial,*

*Ahmad Farhat, Chatham Financial*

We apply vine copulas with GARCH marginals to the problem of capturing asset dependence and tail dynamics for currencies and commodities commonly found in portfolios of global corporates. This approach allows for a more realistic description of both the pairwise dependence of asset prices and the large movements of a

given asset, i.e. fat tails. We compare the cashflow-at-risk (CFaR), expected tail loss (ETL), back-test performance and optimal hedging order for this approach to both the traditional Gaussian model and the analogous t-Copula fit for a pair of hypothetical corporate portfolios. The results suggest that while more traditional models may be adequate to capture either tail-behavior or dependence adequately, vine-GARCH models more consistently capture both dependence and tail risk. This leads to more reliable estimation of the risk held in a portfolio and thus to better hedge performance. Finally, we describe the set of diagnostics used to measure the relative quality of the developed models for purposes of model selection.

## **Dependence Modeling in Finance, Insurance and Environmental Science Munich, May 17 – 19, 2016**

In our complex and interrelated world, taking care of dependence is a key task for success in stochastic modeling. The copula framework takes a pivotal role in this area. The conference will bring together researchers from different methodological view points and application areas to provide an up to date forum on dependence modeling and their challenges in view of big data issues. This conference is a continuation of workshops held in Delft (2007, 2008), Oslo (2009), Munich (2011), and Beijing (2014). While the first workshops were focused on vine copulas (vine-copula.org), the theme of the last workshop was much broader. This workshop is intended to continue with this broader view.

### **Scientific Organizers**

C. Czado and M. Scherer

### **Local Organizers**

B. Haas, M. Killiches and D. Mueller

### **Website**

<http://www.mathfinance.ma.tum.de/copulas2016/>